ALTERNATIVE SUBBAND SIGNAL STRUCTURES FOR COMPLEX MODULATED FILTER BANKS WITH PERFECT RECONSTRUCTION

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ABSTRACT

In this paper, two critically sampled perfect reconstruction complex modulated filter banks, used with complex-valued input signals, are compared. The modified discrete Fourier transform filter bank (MDFT-FB) and the recently introduced exponentially modulated filter bank (EMFB) have different approach for critical sampling and they use different channel stacking arrangement. It is shown that with certain special modifications, the EMFB system can provide exactly the same subband signal values for the subband processing as the MDFT-FB system. An intuitive and simple spectral interpretation for these modifications and alternative spectral structure of the subband signals are also given.

1. INTRODUCTION

$M$-channel cosine modulated filter banks (CMFBs) satisfying perfect reconstruction (PR) property were originally designed for subband processing for real-valued signals [1]. On the other hand, $2M$-channel complex modulated filter banks are basic elements in subband processing systems for complex-valued signals. In traditional discrete Fourier transform (DFT) filter banks the PR property is not feasible in practice [1]. The generalized DFT (GDFT) allows more general representation for the channel filters, because arbitrary time and frequency origins together with longer prototype filters can be used [2]. GDFT filter banks with real-valued input signals are used for subband adaptive filtering in [3] [4].

In this paper, the modified DFT filter banks (MDFT-FBs) [5] and the recently introduced exponentially modulated filter banks (EMFBs) [6], used for subband processing for complex-valued input signals, are studied. They both can be considered to be particular kind of GDFT filter banks. A prototype filter designed for an $M$-channel PR CMFB can be directly used for a $2M$-channel EMFB and MDFT-FB [7]. All the resulting $2M$ channel filters are highly selective and they are equally spaced. Moreover, the EMFB and MDFT-FB are uniform, critically sampled, and they satisfy the PR condition.

However, the critical sampling is accomplished differently and their channel stacking arrangements are different. The EMFB uses odd-stacked channel filters and the MDFT-FB even-stacked channel filters. Due to the definition, all channel filters are purely complex-valued in EMFBs, but in MDFT-FBs the first is real and the $M$th is imaginary. Typical ideal responses of the prototype filter and channel filters with different channel stacking arrangements are shown in Fig. 1.

![Fig. 1](image-url)

Fig. 1. a) Typical ideal response of the prototype filter. b) Even-stacked and c) odd-stacked channel arrangement.

This paper analyzes EMFBs and MDFT-FBs based on the subband signals they use for the subband processing. In Section 2, both of these filter bank systems are reviewed.
Section 3 shows that by using certain special modifications in the EMFB system, exactly the same subband signal values can be obtained as in the MDFT-FB system. A simple and intuitive spectral interpretation is also given for these modifications. In Section 4, the modified EMFB system that provides exactly the same subband signal values as the MDFT-FB system is introduced.

2. OVERVIEW OF EMFB AND MDFT-FB

2.1. EMFB

The EMFB concept is very closely related to the modulated complex lapped transform (MCLT), which is 2x oversampled system for processing real-valued input signals [9]. In both approaches, the synthesis filters \( f_k(n) \) can be generated from a linear-phase lowpass finite impulse response (FIR) prototype filter \( h_p(n) \) by using the complex exponential modulation sequences as follows

\[
f_k(n) = \sqrt{\frac{2}{M}} h_p(n) \exp \left( j \pi \left( k + \frac{1}{2} \right) \left( n + \frac{M + 1}{2} \right) \right),
\]

where \( k = 0, 1, \ldots, 2M - 1 \) and \( n = 0, 1, \ldots, N \). The MCLT uses only \( M \) first channels, because it is used to process real-valued signals. Moreover, the filter order for the MCLT is constrained to be \( N = 2M - 1 \), whereas for the EMFB it is \( N = 2KM - 1 \), with \( K \) being a positive integer.

The analysis filters \( h_k^*(n) \) are time-reversed and complex conjugated versions of the corresponding synthesis filters i.e., \( h_k^*(n) = f_k^*(N - n)^* = -j(1)^{k+K} f_k^*(n) \). All the analysis and synthesis filters are linear-phase filters and their impulse responses are purely complex-valued. Moreover, when the analysis and the corresponding synthesis filters are cascaded, the resulting filters have linear phase.

Figure 2 shows the EMFB system, where the analysis filter bank decomposes a complex-valued high-rate signal into low-rate subband signals. In the figure, double lines refer to complex-valued signals. There are \( 2M \) channels, two times as much as the downsampling factor, but the overall sample rate is preserved because only real parts are used in the subband processing unit. The synthesis filter bank can reconstruct the complex-valued output signal perfectly from the real-valued subband signals as verified in [10]. The resulting output signal is a delayed version of the input signal and the total system delay is equal to the filter order \( N \).

2.2. MDFT-FB

The MDFT-FB is derived from a DFT filter bank with oversampling factor of 2 by introducing several modifications in subbands [7]. The impulse responses of the analysis filters are generated as follows

\[
h_k(n) = \sqrt{2} h_p(n) \exp \left( j \frac{2\pi k}{2M} \left( n + \frac{N}{2} \right) \right),
\]

where \( k = 0, 1, \ldots, 2M - 1 \) and \( n = 0, 1, \ldots, 2KM - 1 \). The synthesis filters are time-reversed and complex conjugates of the corresponding analysis filters, which in this case means that they are identical i.e., \( f_k(n) = h_k(N - n)^* = h_k(n) \). Due to the definition (2), the 0th channel filter is purely real and \( M \)th channel filter is purely imaginary. All the channel filters have linear phase which is an important feature for subband image coding applications.

The subband system of Fig. 3 is not suitable since it does not provide the desired alias cancellation. By using the modifications shown in Fig. 4, all odd aliasing terms are cancelled [7]. All the even alias terms can be cancelled by using a properly designed prototype filter. The key idea is to use a two-step downsampling and upsampling for each subband in the analysis and synthesis filter bank. After a complex-valued input signal is filtered using \( 2M \) analysis filters the complex-valued subband signals are first downsampled by a factor of \( M \). Each of these \( 2M \) subbands is decomposed into two (even/odd) polyphase components that are complex-valued. This is achieved when signals are downsampled by a factor of 2 with and without a unit delay. Critical sampling is obtained by taking the real part of one polyphase component and the imaginary part of the other polyphase component in each subband and alternating this from one subband to the next as presented in Fig. 4. In the synthesis filter bank, similar modifications are performed. The price to be paid for these modifications is that the total system delay increases from \( N \) to \( N + M \).

3. MODEL FOR MDFT-FB SYSTEM

From now on the scaling factor is embodied into the prototype filter to simplify the expressions. Moreover, it is assumed that EMFB and MDFT-FB channel filters are scaled in such a manner that they have same levels.

For EMFB, the \( k \)th analysis filter can also be defined as follows

\[
h_k(n) = h_p(n) \exp \left( -j \left( \frac{\pi}{M} \left( k + \frac{1}{2} \right) \left( N - n + \frac{N}{2} \right) + \phi_k \right) \right),
\]

where \( \phi_k = (k + \frac{1}{2})(2K + 1)\pi \). This form can be easily
The above models include frequency shifts by \( \pm \frac{\pi}{2M} \) and channel dependent phase terms (conversion between odd- and even-stacked arrangements). Now we can write the relation between the channel filters

\[
H_k(z) = H_k^c(z e^{j\pi k/2 M}) e^{j\theta H_k}
\]

and

\[
F_k(z) = F_k^c(z e^{j\pi k/2 M}) e^{j\phi_k}.
\]

The above models include frequency shifts by \( \pm \frac{\pi}{2M} \) (for conversion between odd- and even-stacked arrangements) and channel dependent phase terms \( \theta H_k \) and \( \phi_k \).

In order to mimic the MDFT-FB system, some modifications are needed, because the real part of one polyphase component and the imaginary part of the other component are taken in each subband and this alternates from one subband to the next. The complete model is shown in Fig. 5.

When considering the \( k \)th subband signal, the downsampling by only a factor of \( M \) leaves gaps to the spectrum. This is because the bandwidth of the channel filters is \( \frac{\pi}{M} \) and the periodicity of spectral replicas due to downsampling is \( 2\pi \). However, these gaps are useless if the real part is taken directly, because the resulting mirror image spectra appear at the same positions as the original spectra. If a frequency shift to the right by \( \frac{\pi}{2} \) is performed before taking the real part then these mirror image spectra fill the gaps. Later on, the spectrum is shifted back to its original place. Complex modulators \( e^{j\alpha_k(n)} \) and \( e^{-j\alpha_k(n)} \), where

\[
\alpha_k(n) = \begin{cases} 
\frac{\pi}{2} n, & k \text{ even} \\
\frac{\pi}{2} n + \frac{\pi}{2}, & k \text{ odd}, 
\end{cases}
\]

can be used for these purposes. The output signal values from the analysis part of Fig. 5 are the same as the signal values before upsampling by \( M \) in the MDFT-FB system of Fig. 4. This means that the two polyphase component signals in MDFT-FB are combined to one complex-valued signal in the structure of Fig. 5. If needed, exactly the same subband signals as in MDFT-FB can obtained by simple processing as discussed in Section 5.

### 4. MODIFIED EMFB SYSTEM

We do not have an efficient implementation for the shifted channel filters needed in the previous approach. Fortunately, the original EMFB system can be used, if slightly different modifications are utilized as shown in Fig. 6.

Instead of using shifted channel filters, the input signal is here properly shifted in frequency. Moreover, the scaling by \( e^{j\theta H_k} \) is done before the analysis filters. Then efficient implementation for the analysis EMFB system is used and the resulting real-valued subband signals are modulated by \( e^{j\beta_k(n)} \), where

\[
\beta_k(n) = -\frac{\pi}{2} n + (-1)^k \frac{\pi}{2} k.
\]
In this case the operation of taking the real part can be performed directly because EMFB has odd-stacked channel arrangement and the mirror image spectrum due to the operation automatically fill the available gaps. After the modulation by \( e^{\pm j\beta_k(n)} \), the complex-valued subband signals are exactly the same as in Fig. 5. In the synthesis side, the signals are demodulated by \( e^{-j\beta_k(n)} \) and filtered with the synthesis filters. The output signal is scaled by \( e^{j\theta_0} \) and demodulated back to its original place.

5. CONCLUDING REMARKS

Based on the MDFT-FB and EMFB structures, we can consider different subband signal formats. To simplify the spectral interpretations, we assume here that the channel filters have sharp transition bands and the passband width is \( \frac{\pi}{M} \):

- **EMFB** (Fig. 2 for odd-stacking or Fig. 6 without \( e^{\pm j\beta_k(n)} \)): Before taking the real part, the passband is in the range \([0, \pi]\) for even channels and \([-\pi, 0]\) for odd channels. The operation of taking the real part produces mirror image spectrum on the other side.

- **Modified EMFB** (Fig. 6): The modulation operation moves the passband to be centered at 0 for even channels and at \(-\pi\) for odd channels.

- **MDFT-FB** (Fig. 4): As a model, the MDFT channel signals can be obtained from the modified EMFB channel signals through a length-2 moving-average filter and downsampling by 2. This causes a partly attenuated image to alias on top of the original passband.

We can conclude that in the EMFB and the modified EMFB, the ideal subband signal spectrum appears in less distorted form; here the distortion is due to overlapping of the (in practice) wide transition bands of the original subband and its image.

It is important to notice that the \( e^{\pm j\beta_k(n)} \) sequences generate only values \( \pm 1 \) and \( \pm j \), so the output samples of the analysis part of Fig. 6 are purely real or purely imaginary, and the rate of non-zero real/imaginary samples is exactly the same in the modified EMFB and the MDFT-FB structures. Also the sample values are exactly the same, they are only arranged in different ways to the subband signals. Therefore, for example, straightforward quantization of the subband signal values would result in exactly the same synthesized signals in the modified EMFB and the MDFT-FB cases. With these observations in mind, it remains as a topic for future studies to find the most suitable subband signal structure for each application, considering also the importance of the nonlinear-phase properties. Another topic is the implementation complexity of the alternative structures. The EMFB approach has the benefit that it can be efficiently implemented using cosine and sine modulated filter banks.

6. REFERENCES


